

# Bequests and the Intergenerational Degree of Altruism\*

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## 1 Introduction

This paper deals with the overlapping generations (OLG) model with production and altruistic agents. We adopt Barro's (1974) formalization of the bequest motive: parents care about their children's welfare, and possibly leave them a bequest. Importantly, the bequest is restricted to be non-negative. Barro's debt neutrality theorem hinges on the existence of an equilibrium with positive bequests. Indeed, if bequests are constrained the Ricardian equivalence result does not hold and there are important effects associated with public debt.

Abel (1987) and Weil (1987) show that bequests are positive if and only if the intensity of altruism is sufficiently strong. Nevertheless, their characterization of equilibrium rests on the assumption of existence, uniqueness and stability of the steady state of the Diamond (1965) model.

It could be deduced from this result that an increase in the degree of altruism leads to an increase in the stationary level of bequests. Since Abel (1987) and Weil (1987), it is a widespread opinion that existence and uniqueness of a stable steady state in Diamond's model are sufficient to ensure a such property.

The aim of this paper is to investigate the relevance of this opinion.

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We begin to show that an increase in the degree of altruism leads to a decrease in the stationary level of bequests if the concavity of the production function is strong enough at the modified golden rule steady state.

In this case, intuitively, a small increase in the modified golden rule steady state implies a large decrease in the interest factor but a small increase in the level of savings of parents. Hence, this worsening of their saving income is such that parents bequeath less although their bequest motive is stronger.

Finally, we provide an example showing that assumptions made by Abel (1987) and Weil (1987) are not sufficient to rule out this case.

This paper is organized as follows. Section 2 sets up the OLG model with production and altruistic preferences. Section 3 contains our results. Conclusions are gathered in Section 4.

## 2 The model

Consider a perfectly competitive economy which extends over infinite discrete time. Population consists of altruistic agents who are identical within as well as across generations and live for two periods. Young altruists supply a fixed amount of labor, receive the market wage  $w_t$ , inherit  $x_t$ , consume  $c_t$ , and save  $s_t$ . When old they consume  $d_{t+1}$ , a part of the proceeds of their savings, and bequeath the remainder,  $x_{t+1}$ , to their child. Both periods' consumption  $(c_t, d_{t+1})$  are normal goods. Let  $R_{t+1}$  be the (real) factor of interest. Thus, agents face the constraints :

$$x_t + w_t = c_t + s_t \quad \text{and} \quad R_{t+1}s_t = d_{t+1} + x_{t+1} \quad (1)$$

$$\forall t \quad x_t \geq 0 \quad (2)$$

where negative bequests are prohibited by condition (2).

Parents are altruistic with respect to their child. They care about their child's welfare by weighting their child's utility in their own utility function. Indeed, an altruistic agent who inherits  $x_t$  has a utility function  $V_t$  of the form :

$$V_t(x_t) = \max_{c_t, d_{t+1}, x_{t+1}} \{U(c_t, d_{t+1}) + \beta V_{t+1}(x_{t+1})\}$$

where  $U(c, d)$  is his life cycle utility and  $V_{t+1}(x_{t+1})$  denotes the utility of his descendant when he inherits  $x_{t+1}$ . The intergenerational degree of altruism is represented by  $\beta \in (0, 1)$ .

The utility function  $U(c, d)$  is strictly concave, twice continuously differentiable over  $\mathbb{R}_+^2$  and satisfies  $U'_c(c, d) > 0$ ,  $U'_d(c, d) > 0$ ,  $\lim_{c \rightarrow 0} U'_c(c, d) = +\infty$ , and  $\lim_{d \rightarrow 0} U'_d(c, d) = +\infty$ .

An altruist who inherits  $x_t$  solves  $V_t(x_t)$  subject to (1) and (2).

The saving function  $s_t \equiv s(w_t + x_t, x_{t+1}, R_{t+1})$  is given by

$$s_t = \arg \max_s U(w_t + x_t - s, R_{t+1}s - x_{t+1})$$

The output produced at time  $t$  using capital  $K_t$  and labor  $L_t$ , is governed by a production function with constant returns to scale  $F(K_t, L_t) = L_t f(k_t)$  where  $k_t = K_t/L_t$ . The function  $f$  is concave, strictly monotonic, and twice continuously differentiable for  $k > 0$ . Capital fully depreciates after one period. Competition in the markets for capital and labor services ensures that each factor is paid its marginal product :

$$w_t = f(k_t) - k_t f'(k_t) \quad \text{and} \quad R_t = f'(k_t) \quad (3)$$

Capital stock of period  $t+1$  is financed by saving of period  $t$  *i.e.*  $k_{t+1} = s_t$ .

### 3 The results

Recently, we have shown in Thibault (2000) that, unlike the Diamond model (see Galor and Ryder, 1989), the OLG model with altruistic agents always experiences a non-trivial steady state. we can distinguish two kinds of steady state whether bequests are positive or nil.

When we consider the standard case where Diamond economy has a unique and stable steady state  $(k^*, R^*)$ , the condition for positive bequests is the well-known condition exhibited by Abel (1987) and Weil (1987) :

$$\beta > 1/R^*$$

In such a case there exists a threshold value of the degree of altruism above which altruists leave bequest. According to conventional wisdom the bequest left by an agent increases w.r.t. his degree of altruism above this threshold. A widespread opinion actually is that the existence and the uniqueness of a stable steady state in the Diamond model are sufficient to prevent the converse and rather counterintuitive result (see Abel (1987 p. 1042)<sup>1</sup>).

When the model has a steady state with positive bequests, the stationary capital stock is the modified golden rule steady state capital  $\hat{k} = f'^{-1}(1/\beta)$ . Then, according to (3), stationary wage and interest rate are  $\hat{w} = f(\hat{k}) - \hat{k}/\beta$  and  $\hat{R} = 1/\beta$ , respectively. We begin our study by deriving

<sup>1</sup> Abel (1987) : "To avoid any complications that may arise from multiple locally stable steady states, I follow Weil and assume that there is a unique locally stable steady state."

a necessary and sufficient condition under which the degree of altruism is negatively related to the level of bequests.

**Proposition 1** *Let  $\hat{s}'_i = s'_i(\hat{w} + x, -x, \hat{R})$  the derivative of  $s$  w.r.t. its  $i$ -th argument evaluated at the modified golden rule steady state.*

*An increase in the degree of altruism results in a decrease in the stationary level of bequests if and only if  $|f''(\hat{k})| > 1/[\hat{k}\hat{s}'_1 - \hat{s}'_3] > 0$ .*

**Proof.** Let  $w(k) = f(k) - kf'(k)$  and  $R(k) = f'(k)$ .

Since  $\hat{k} = s(w(\hat{k}) + x, -x, R(\hat{k}))$  and using the implicit function theorem,  $x$  can be written as a function of  $\hat{k}$ ,  $R(\hat{k})$  and  $w(\hat{k})$ . Hence, according to (3), there exists  $\Phi$  such that:  $x = \Phi(\hat{k}, f(\hat{k}), f'(\hat{k}))$ .

Since  $\hat{k} = s(w(\hat{k}) + x, -x, R(\hat{k}))$  we have:  $dx/d\hat{k} = [1 - (w'\hat{s}'_1 + R'\hat{s}'_3)]/[\hat{s}'_1 - \hat{s}'_2]$ . Since  $\hat{k} = f^{-1}(\frac{1}{\beta})$  is increasing w.r.t.  $\beta$ ,  $dx/d\beta$  and  $dx/d\hat{k}$  have the same sign. Since  $c$  and  $d$  are normal goods we have  $\hat{s}'_1 > 0$  and  $\hat{s}'_2 < 0$ . Therefore  $\partial x/\partial\beta$  has the sign of  $1 - (w'\hat{s}'_1 + R'\hat{s}'_3)$  i.e.  $1 + f''(\hat{k})(\hat{k}\hat{s}'_1 - \hat{s}'_3)$ . Hence, If  $\hat{s}'_3 > \hat{k}\hat{s}'_1$  then  $\partial x/\partial\beta$  is positive. If  $\hat{s}'_3 < \hat{k}\hat{s}'_1$  then  $\partial x/\partial\beta$  is negative if and only if  $|f''(\hat{k})| > 1/[\hat{k}\hat{s}'_1 - \hat{s}'_3]$ .  $\square$

Note that  $\hat{s}'_3 < \hat{k}\hat{s}'_1$  is a necessary condition so that the degree of altruism is negatively related to the level of bequests. Since both periods' consumptions are normal goods we have  $\hat{s}'_1 > 0$ . Then, the previous necessary condition is satisfied if  $\hat{s}'_3$  is negative. We focus now on the Cobb-Douglas case in which  $\hat{s}'_3$  is negative to give an economic intuition of proposition 1.

**Corollary 1** *Let the life cycle utility function be Cobb-Douglas.<sup>2</sup>*

*The degree of altruism is negatively related to the level of bequests if and only if  $|f''(\hat{k})|$  is sufficiently high.*

**Proof.** If  $U(c_t, d_{t+1}) = \ln c_t + \gamma \ln d_{t+1}$  we have:  $s(w_t + x_t, -x_{t+1}, R_{t+1}) = [\gamma/(1+\gamma)](w_t + x_t) + x_{t+1}/[(1+\gamma)R_{t+1}]$ . Hence,  $s'_1 > 0$  and  $s'_3 \leq 0$ . Since  $\hat{s}'_3 < \hat{k}\hat{s}'_1$ , according to proposition 1,  $\partial x/\partial\beta$  is negative if and only if  $|f''(\hat{k})| > 1/[\hat{k}\hat{s}'_1 - \hat{s}'_3]$ . According to (3) and since  $x = \Phi(\hat{k}, f(\hat{k}), f'(\hat{k}))$ ,  $\hat{s}'_i$  depends on  $\hat{k}$ ,  $f(\hat{k})$ , and  $f'(\hat{k})$ . Hence, there exists  $\Psi$  such that  $\partial x/\partial\beta$  is negative if and only if  $|f''(\hat{k})| > \Psi(\hat{k}, f(\hat{k}), f'(\hat{k})) > 0$ .  $\square$

An increase in the degree of altruism results in a decrease in the stationary level of bequests if the concavity of the production function is strong enough at the modified golden rule steady state. Intuitively, an infinitesimal increase in the modified golden rule steady state leads to decrease the interest factor but does not significantly change the stationary level of saving of parents. Hence, when the concavity of the production function is strong enough at the modified golden rule steady state the worsening of their sa-

<sup>2</sup> Precisely, this corollary is valid for all life cycle utility function satisfying  $\hat{s}'_3 < \hat{k}\hat{s}'_1$ .

ving income,  $R_s$  is such that parents bequeath less although their bequest motive is stronger.<sup>3</sup>

Note that our argument is not based on a small variation in  $\beta$  since such variation may have, depending on the properties of  $f$ , a strong effect on the saving behavior. On the contrary, a small increase in the capital stock  $k$  implies an infinitesimal increase in saving  $s$  and simplifies the analysis of the behavior of altruist.

Indeed, since there is complete depreciation of the capital stock, then second period income equals  $kf'(k)$ . Whether this increases or decreases as  $k$  goes to the golden rule capital stock depends on the production function.<sup>4</sup> Clearly with “ $Ak$ ” production this expression increases but with a production function whose gradient quickly approaches zero as  $\beta$  goes to one, this expression will tend to zero. Thus an increase in saving (altruism) will actually reduce second period consumption (bequests).<sup>5</sup>

From corollary 1, we can construct a production function satisfying the Inada conditions and the assumptions of Abel (1987) and Weil (1987) such that an increase in the degree of altruism results in a decrease in the stationary level of bequests.

**Example 1** Consider an economy described by the Cobb-Douglas utility function  $U(c, d) = \frac{1}{2} \ln c + \frac{1}{2} \ln d$  and the production function :

$$f(k) = \begin{cases} 2 + 5k - k \ln k & \text{for } 0 < k \leq 4 \\ \left(\frac{13}{2} + 3 \ln 2\right) k - \frac{5}{2} k \ln k - 4 & \text{for } 4 < k \leq 6 \\ \left(6 + \frac{5}{2} \ln 2 - \frac{1}{2} \ln 3\right) k - 2k \ln k - 1 & \text{for } 6 < k \leq 12 \\ Ak^{\frac{1}{3}} + B & \text{for } k > 12 \end{cases}$$

where  $A = 12^{\frac{2}{3}}(12 - \frac{9}{2} \ln 2 - \frac{15}{2} \ln 3)$  and  $B = 36 \ln 2 + 60 \ln 3 - 73$ .

The stationary level of bequests  $\hat{x}$  is represented as a function of  $\beta$  in figure 1.

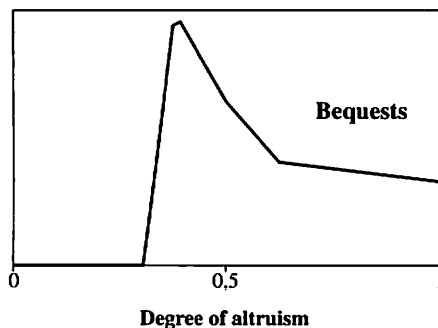


Figure 1 : Bequests and the intergenerational degree of altruism

<sup>3</sup> Although their wage rate has also increased, this additional income is used to increase their first period consumption.

<sup>4</sup> When  $\beta$  tends to one,  $\hat{k}$  goes to  $f'^{-1}(1)$  the golden rule capital stock.

<sup>5</sup> I am grateful to a referee for making this observation.

This example shows that Abel (1987) and Weil (1987) assumptions are fully compatible with the concavity assumption which ensures that degree of altruism is negatively related to the level of bequests.

## 4 Conclusion

We have considered an overlapping generations model with production and altruistic agents initially developed by Barro (1974). We have shown that, contrary to the intuition of Abel (1987) and Weil (1987), an increase in the degree of altruism can result in a decrease in the stationary level of bequests even if the Diamond model has a unique and stable steady state. To rule out this counterintuitive case, an assumption on the curvature of the production function is necessary.

## Appendix A : Computational details of Example 1

$f$  is continuous, differentiable,  $f'$  is continuously, positive and  $f''$  is negative. Indeed :

$$f'(k) = R(k) = \begin{cases} 4 - \ln k & \text{if } 0 < k \leq 4 \\ 4 + 3 \ln 2 - \frac{5}{2} \ln k & \text{if } 4 < k \leq 6 \\ 4 + \frac{5}{2} \ln 2 - \frac{1}{2} \ln 3 - 2 \ln k & \text{if } 6 < k \leq 12 \\ \frac{A}{3} k^{-2/3} & \text{if } k > 12 \end{cases}$$

$$f''(k) = \begin{cases} -\frac{1}{k} & \text{if } 0 < k \leq 4 \\ -\frac{5}{2k} & \text{if } 4 < k \leq 6 \\ -\frac{2}{k} & \text{if } 6 < k \leq 12 \\ -\frac{2A}{9} k^{-5/3} & \text{if } k > 12 \end{cases} \quad \text{and } w(k) = \begin{cases} 2 + k & \text{if } 0 < k \leq 4 \\ \frac{5}{2} k - 4 & \text{if } 4 < k \leq 6 \\ 2k - 1 & \text{if } 6 < k \leq 12 \\ \frac{2A}{3} k^{1/3} + B & \text{if } k > 12 \end{cases}$$

Hence  $w(k)$  and  $R(k)$  are continuous functions.

With  $U(c_t, d_{t+1}) = \frac{1}{2} \ln c_t + \frac{1}{2} \ln d_{t+1}$  we have :

$$s(w_t + x_t, -x_{t+1}, R_{t+1}) = \frac{1}{2} \left( w_t + x_t + \frac{x_{t+1}}{R_{t+1}} \right)$$

Then the saving of the economy without bequest  $s_t^D$  satisfies :  $s_t^D = \frac{1}{2} w_t$ .

The dynamics of the Diamond model are  $k_{t+1} = \Phi(k_t)$ .

$$\Phi(k_t) = \begin{cases} 1 + \frac{k_t}{2} & \text{if } 0 < k_t \leq 4 \\ \frac{5}{4} k_t - 2 & \text{if } 4 < k_t \leq 6 \\ k_t - \frac{1}{2} & \text{if } 6 < k_t \leq 12 \\ \frac{A}{3} k_t^{1/3} + \frac{1}{2} B & \text{if } k_t > 12 \end{cases}$$

This implies that the Diamond model experiences a unique and globally stable (non trivial) steady-state equilibrium  $k^D$  where  $k^D = 2$ .

From corollary 1 and after computation, we have :  $\hat{x} > 0 \Leftrightarrow \beta > \frac{1}{4 - \ln 2}$ .

$$\text{Since } \hat{k} = \hat{s} = \frac{1}{2} \left\{ \hat{w} + \left( 1 + \frac{1}{\hat{R}} \right) \hat{x} \right\} \text{ we have } \hat{x} = \begin{cases} 0 & \text{if } \beta \leq \frac{1}{4 - \ln 2} \\ \frac{2\hat{k} - \hat{w}}{1 + \beta} & \text{if } \beta > \frac{1}{4 - \ln 2} \end{cases}$$

We have :

$$\hat{k} = \begin{cases} \exp \left\{ 4 - \frac{1}{\beta} \right\} & \text{if } \beta \in \left] 0, \frac{1}{4 - \ln 2} \right] \\ \exp \left\{ \frac{8}{5} + \frac{6}{5} \ln 2 - \frac{2}{5\beta} \right\} & \text{if } \beta \in \left] \frac{1}{4 - \ln 2}, \frac{1}{4 + \frac{1}{2} \ln 2 - \frac{5}{2} \ln 3} \right] \\ \exp \left\{ 2 + \frac{5}{4} \ln 2 - \frac{1}{4} \ln 3 - \frac{1}{2\beta} \right\} & \text{if } \beta \in \left] \frac{1}{4 + \frac{1}{2} \ln 2 - \frac{5}{2} \ln 3}, 1 \right] \end{cases}$$

Finally we obtain :

$$\hat{x} = \begin{cases} 0 & \text{if } \beta \in \left] 0, \frac{1}{4 - \ln 2} \right] \\ \left[ \exp \left\{ 4 - \frac{1}{\beta} \right\} - 2 \right] / (1 + \beta) & \text{if } \beta \in \left] \frac{1}{4 - \ln 2}, \frac{1}{4 - 2 \ln 2} \right] \\ \left[ 4 - \frac{1}{2} \exp \left\{ \frac{8}{5} + \frac{6}{5} \ln 2 - \frac{2}{5\beta} \right\} \right] / (1 + \beta) & \text{if } \beta \in \left] \frac{1}{4 - 2 \ln 2}, \frac{1}{4 + \frac{1}{2} \ln 2 - \frac{5}{2} \ln 3} \right] \\ 1 / (1 + \beta) & \text{if } \beta \in \left] \frac{1}{4 + \frac{1}{2} \ln 2 - \frac{5}{2} \ln 3}, 1 \right] \end{cases}$$

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